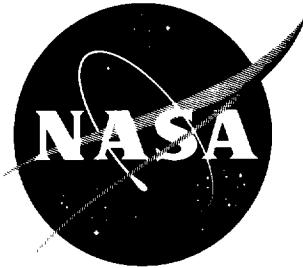


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## TECHNICAL NOTE

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### THE MAGNETIC FIELD OF THE RADIATION BELTS

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# **THE MAGNETIC FIELD OF THE RADIATION BELTS**

by

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## **SUMMARY**

The magnetic fields produced by various types of belts of trapped particles are presented in both tabular and graphical forms.



# THE MAGNETIC FIELD OF THE RADIATION BELTS\*

by

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In this note we shall present in graphical form the magnetic fields produced by various types of belts of trapped particles. The chosen values of the parameters determining the field cover a wide range. It is hoped that, by interpolating the diagrams shown, the distortion of the earth's field can be approximately estimated for actually observed belts without a large amount of computation. References 1-5 discuss the method of computation.

Herein we shall state briefly the formulas used. The number density distribution along an equatorial radius  $r$ :  $N = f_1(r)$  is given by:

$$N = N_0 e^{-g_1^2 z^2} \quad (z < 0 \text{ for the inner part of the belt});$$

$$N = N_0 e^{-g_2^2 z^2} \quad (z > 0 \text{ for the outer part of the belt}),$$

where  $z = (r - r_0)/a$ ,  $a$  is the radius of the earth, and  $r_0$  denotes the distance at which  $N$  attains its maximum value  $N_0$ . Thus we may write

$$N = f_1(r_0, z, g_1, g_2) .$$

The pitch-angle distribution,  $P = f_2(\theta, \alpha)$ , is

$$P = A(\alpha) \sin^{\alpha+1} \theta ,$$

where  $\theta$  denotes the pitch angle,  $\alpha$  is a constant, and  $A(\alpha)$  is a normalization factor.

In the energy spectrum  $N = f_3(E)$ , we shall consider only particles with a particular energy  $E$  (or speed  $v$ ). The functions  $f_1$ ,  $f_2$ , and  $f_3$  determine the electric current intensity at any point in a dipole field. The ring current field  $\Delta F$  is then calculated by a method shown in Reference 2.

\*This report has been published in substantially the same form minus Appendix A, in *J. Geophys. Res.* 67(10):4078-4080, September 1962.

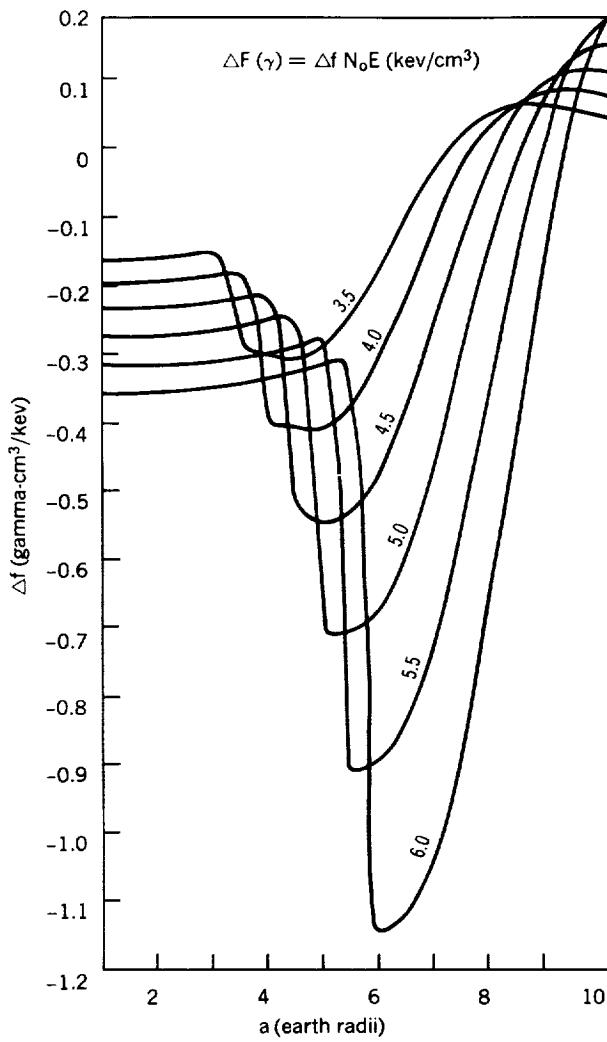


Figure 1(a)—Ring current field  $\Delta f$  for several values of  $r_0$ .

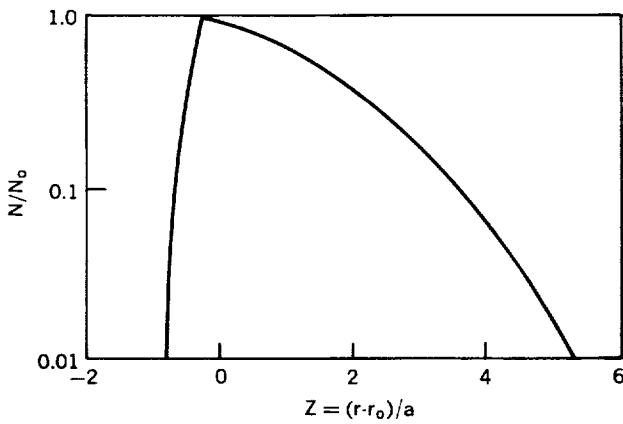


Figure 1(b)—Normalized electron density distribution.

The ring current field  $\Delta F$  is given by:

$$\Delta F = f(r, r_0, g_1, g_2, \alpha, N_0, E) . \quad (1)$$

In the first approximation,  $\Delta F$  is proportional to both  $N_0$  and  $E$ . Therefore, Equation 1 may be rewritten

$$\Delta F = \Delta f N_0 E , \quad (2)$$

where

$$\Delta f = f'(r, r_0, g_1, g_2, \alpha) . \quad (3)$$

We will show  $\Delta f$ , as a function of equatorial radial distance  $r$ , for various sets of  $r_0$ ,  $g_1$ ,  $g_2$ , and  $\alpha$ . The vector  $\Delta F$  is perpendicular to the equatorial plane and its value of  $\Delta F$  is given in gammas, if the units of  $N_0$  and  $E$  are chosen as  $\text{cm}^{-3}$  and  $\text{kev}$ , respectively.

In Figure 1(a)  $\Delta f$  is shown for different values of  $r_0$  (Appendix A). The other parameters  $g_1$ ,  $g_2$ , and  $\alpha$  are chosen in such a way that they are suitable for the quiet-time proton belt (Reference 3):

$$g_1 = 2.990 \quad (N/N_0 = 1/10 \text{ at } z = -0.51),$$

$$g_2 = 0.419 \quad (N/N_0 = 1/10 \text{ at } z = +3.62),$$

$$\alpha = 2.0 .$$

The graph of  $N/N_0$  is given in Figure 1(b).

Figure 2(a) presents  $\Delta f$  for different values of  $g_2$ , namely:

$$g_2 = 2.146 \quad (N/N_0 = 1/100 \text{ at } z = +1) ,$$

$$g_2 = 1.517 \quad (N/N_0 = 1/10 \text{ at } z = +1) ,$$

$$g_2 = 0.759 \quad (N/N_0 = 1/10 \text{ at } z = +2) ,$$

$$g_2 = 0.379 \quad (N/N_0 = 1/10 \text{ at } z = +4) .$$

The graph of  $N/N_0$  for these four cases is shown in Figure 2(b). The other parameters are chosen as follows:

$$r_0 = 3.2a$$

$$g_1 = 2.146 \quad (N/N_0 = 1/100 \text{ at } z = -1) ,$$

$$\alpha = 2.0$$

In Figure 3(a)  $\Delta f$  is given for different values of  $\alpha$ ; the graph of  $P/A$  for various values of  $\alpha$  is given in Figure 3(b). The other parameters are chosen as follows:

$$r_0 = 3.2a$$

$$g_1 = 2.628 \quad (N/N_0 = 1/1000 \text{ at } z = -1) ,$$

$$g_2 = 0.379 \quad (N/N_0 = 1/10 \text{ at } z = +4) .$$

The tabulated data from which Figures 1(a), 2(a), and 3(a) are plotted are presented in Appendix A.

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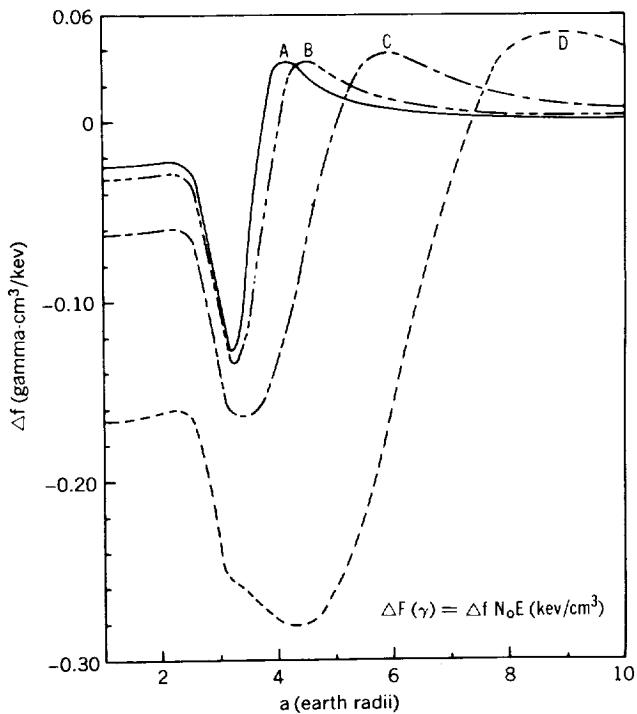


Figure 2(a)—Ring current field  $\Delta f$  for several values of outer slope. The values of  $g_2$  for curves A, B, C, and D are 2.146, 1.517, 0.759 and 0.379 respectively.

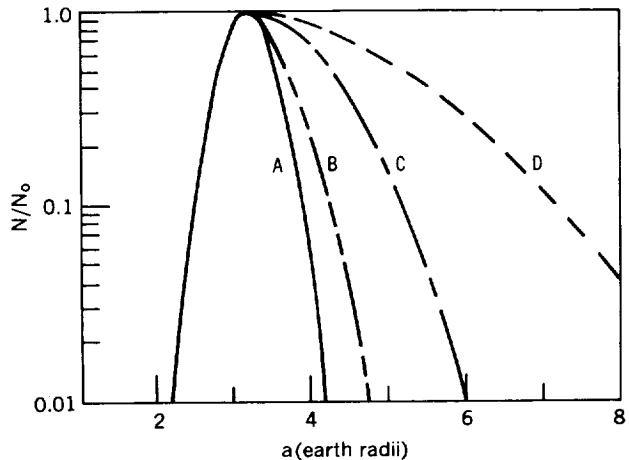


Figure 2(b)—The normalized electron density distribution. The curves A, B, C, and D are defined in Figure 4a.

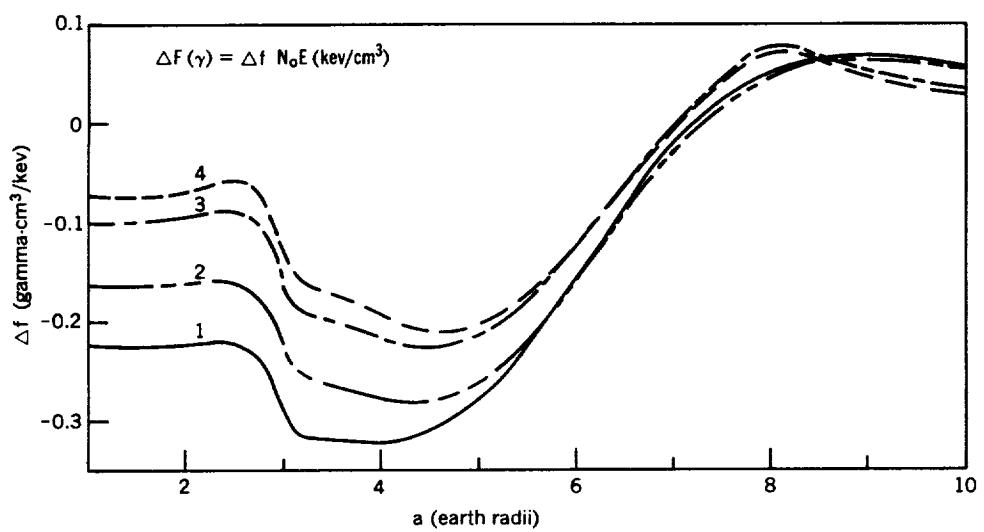


Figure 3(a)—Ring current field  $\Delta f$  for several  $\alpha$ .

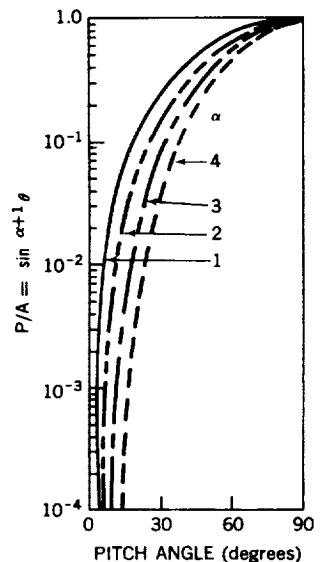


Figure 3(b)—Normalized pitch angle distribution for various  $\alpha$ .

4. Beard, D. B., "Self-Consistent Calculation of the Ring Current," *J. Geophys. Res.* 67(9):3615-3616, August 1962.
5. Akasofu, S.-I., "On a Self-Consistent Calculation of the Ring Current Field," *J. Geophys. Res.* 67(9): 3617-3618, August 1962.

## **Appendix A**

### **The Ring Current Field $\Delta f$ as a Function of $r$ , $g_2$ , and $\alpha$**

Table A1

The Value of  $\Delta F$  as a Function of  $r_0$ .

r (earth radii)	Ring current field (gamma)									
	$r_0 = 1.5a$	$r_0 = 2.0a$	$r_0 = 2.5a$	$r_0 = 3.0a$	$r_0 = 3.5a$	$r_0 = 4.0a$	$r_0 = 4.5a$	$r_0 = 5.0a$	$r_0 = 5.5a$	$r_0 = 6.0a$
1.0	-0.0578	-0.0779	-0.1020	-0.1296	-0.1604	-0.1952	-0.2334	-0.2744	-0.3169	-0.3576
1.1	-0.0577	-0.0778	-0.1019	-0.1295	-0.1602	-0.1950	-0.2332	-0.2743	-0.3167	-0.3574
1.2	-0.0576	-0.0777	-0.1017	-0.1294	-0.1600	-0.1948	-0.2330	-0.2741	-0.3166	-0.3572
1.3	-0.0574	-0.0775	-0.1016	-0.1292	-0.1597	-0.1946	-0.2328	-0.2738	-0.3163	-0.3570
1.4	-0.0572	-0.0773	-0.1014	-0.1291	-0.1594	-0.1943	-0.2328	-0.2736	-0.3161	-0.3568
1.5	-0.0576	-0.0771	-0.1012	-0.1289	-0.1591	-0.1940	-0.2323	-0.2733	-0.3158	-0.3565
1.6	-0.0591	-0.0769	-0.1010	-0.1287	-0.1587	-0.1937	-0.2319	-0.2730	-0.3156	-0.3562
1.7	-0.0614	-0.0766	-0.1007	-0.1284	-0.1583	-0.1933	-0.2316	-0.2727	-0.3152	-0.3559
1.8	-0.0640	-0.0766	-0.1004	-0.1281	-0.1578	-0.1928	-0.2312	-0.2723	-0.3149	-0.3556
1.9	-0.0654	-0.0773	-0.1000	-0.1278	-0.1572	-0.1924	-0.2308	-0.2719	-0.3145	-0.3552
2.0	-0.0662	-0.0794	-0.0996	-0.1274	-0.1566	-0.1918	-0.2303	-0.2715	-0.3141	-0.3548
2.1	-0.0671	-0.0836	-0.0991	-0.1270	-0.1559	-0.1912	-0.2297	-0.2710	-0.3136	-0.3543
2.2	-0.0683	-0.0898	-0.0988	-0.1265	-0.1552	-0.1906	-0.2292	-0.2705	-0.3131	-0.3539
2.3	-0.0696	-0.0961	-0.0991	-0.1259	-0.1543	-0.1898	-0.2285	-0.2699	-0.3126	-0.3534
2.4	-0.0709	-0.0992	-0.1010	-0.1253	-0.1534	-0.1890	-0.2278	-0.2693	-0.3120	-0.3528
2.5	-0.0723	-0.1002	-0.1061	-0.1246	-0.1523	-0.1881	-0.2270	-0.2686	-0.3114	-0.3522
2.6	-0.0737	-0.1017	-0.1156	-0.1240	-0.1512	-0.1872	-0.2262	-0.2678	-0.3107	-0.3515
2.7	-0.0751	-0.1033	-0.1288	-0.1237	-0.1502	-0.1861	-0.2253	-0.2870	-0.3099	-0.3508
2.8	-0.0764	-0.1051	-0.1415	-0.1247	-0.1498	-0.1849	-0.2243	-0.2661	-0.3091	-0.3501
2.9	-0.0776	-0.1070	-0.1472	-0.1288	-0.1517	-0.1836	-0.2231	-0.2651	-0.3083	-0.3493
3.0	-0.0786	-0.1089	-0.1484	-0.1389	-0.1590	-0.1822	-0.2219	-0.2641	-0.3073	-0.3484
3.1	-0.0794	-0.1107	-0.1502	-0.1569	-0.1761	-0.1807	-0.2206	-0.2629	-0.3063	-0.3474
3.2	-0.0800	-0.1123	-0.1522	-0.1810	-0.2061	-0.1795	-0.2192	-0.2617	-0.3052	-0.3464
3.3	-0.0804	-0.1138	-0.1544	-0.2034	-0.2452	-0.1794	-0.2177	-0.2604	-0.3040	-0.3454
3.4	-0.0804	-0.1150	-0.1567	-0.2130	-0.2807	-0.1831	-0.2160	-0.2589	-0.3028	-0.3442
3.5	-0.0801	-0.1160	-0.1589	-0.2142	-0.2949	-0.1953	-0.2142	-0.2874	-0.3014	-0.3430
3.6	-0.0794	-0.1165	-0.1609	-0.2160	-0.2955	-0.2230	-0.2124	-0.2557	-0.2999	-0.3416
3.7	-0.0784	-0.1167	-0.1626	-0.2183	-0.2969	-0.2700	-0.2110	-0.2539	-0.2983	-0.3402
3.8	-0.0771	-0.1164	-0.1639	-0.2207	-0.2989	-0.3304	-0.2114	-0.2520	-0.2967	-0.3387
3.9	-0.0753	-0.1156	-0.1648	-0.2231	-0.3011	-0.3842	-0.2176	-0.2499	-0.2948	-0.3371
4.0	-0.0732	-0.1143	-0.1652	-0.2252	-0.3031	-0.4054	-0.2367	-0.2476	-0.2929	-0.3353
4.1	-0.0707	-0.1126	-0.1651	-0.2270	-0.3049	-0.4056	-0.2784	-0.2455	-0.2908	-0.3335
4.2	-0.0679	-0.1103	-0.1643	-0.2284	-0.3062	-0.4064	-0.3480	-0.2440	-0.2886	-0.3315
4.3	-0.0648	-0.1075	-0.1628	-0.2292	-0.3067	-0.4078	-0.4361	-0.2453	-0.2862	-0.3294
4.4	-0.0614	-0.1041	-0.1607	-0.2293	-0.3065	-0.4094	-0.5135	-0.2550	-0.2837	-0.3272
4.5	-0.0578	-0.1004	-0.1579	-0.2286	-0.3053	-0.4107	-0.5439	-0.2832	-0.2810	-0.3248
4.6	-0.0540	-0.0961	-0.1543	-0.2271	-0.3031	-0.4115	-0.5437	-0.3431	-0.2785	-0.3223
4.7	-0.0500	-0.0915	-0.1501	-0.2247	-0.2997	-0.4115	-0.5435	-0.4414	-0.2770	-0.3196
4.8	-0.0458	-0.0864	-0.1452	-0.2215	-0.2952	-0.4105	-0.5439	-0.5643	-0.2796	-0.3168
4.9	-0.0416	-0.0811	-0.1397	-0.2173	-0.2896	-0.4084	-0.5443	-0.6710	-0.2939	-0.3137
5.0	-0.0373	-0.0754	-0.1335	-0.2121	-0.2827	-0.4050	-0.5442	-0.7131	-0.3337	-0.3106
5.5	-0.0167	-0.0453	-0.0958	-0.1733	-0.2315	-0.3661	-0.5265	-0.7064	-0.9150	-0.3853
6.0	-0.0002	-0.0168	-0.0532	-0.1190	-0.1601	-0.2916	-0.4658	-0.6720	-0.8990	-1.1490
6.5	+0.0103	+0.0050	-0.0150	-0.0610	-0.0838	-0.1944	-0.3619	-0.5831	-0.8426	-1.1220
7.0	+0.0153	+0.0181	+0.0130	-0.0109	-0.0175	-0.0946	-0.2330	-0.4427	-0.7182	-1.0370
7.5	+0.0162	+0.0237	+0.0290	+0.0243	+0.0296	-0.0106	-0.1050	-0.2757	-0.5335	-0.8692
8.0	+0.0149	+0.0242	+0.0352	+0.0436	+0.0560	+0.0475	-0.0002	-0.1144	-0.3211	-0.6314
8.5	+0.0128	+0.0219	+0.0348	+0.0502	+0.0654	+0.0787	+0.0705	+0.0148	-0.1209	-0.3656
9.0	+0.0106	+0.0187	+0.0312	+0.0487	+0.0641	+0.0889	+0.1073	+0.1000	+0.0365	-0.1200
9.5	+0.0087	+0.0155	+0.0266	+0.0434	+0.0573	+0.0859	+0.1182	+0.1430	+0.1384	+0.0700
10.0	+0.0071	+0.0128	+0.0222	+0.0370	+0.0490	+0.0766	+0.1131	+0.1545	+0.1885	+0.1910

Table A2  
The Values of  $\Delta f$  as a function of  $g_z$ .

r (earth radii)	Ring current field (gamma)				
	$g_z = 2.146$	$g_z = 1.517$	$g_z = 0.759$	$g_z = 0.509$	$g_z = 0.379$
1.0	-0.0245	-0.0320	-0.0638	-0.1075	-0.1663
1.1	-0.0245	-0.0319	-0.0637	-0.1073	-0.1661
1.2	-0.0243	-0.0318	-0.0635	-0.1071	-0.1658
1.3	-0.0242	-0.0317	-0.0633	-0.1068	-0.1655
1.4	-0.0241	-0.0315	-0.0631	-0.1065	-0.1651
1.5	-0.0239	-0.0313	-0.0628	-0.1062	-0.1647
1.6	-0.0237	-0.0311	-0.0625	-0.1058	-0.1642
1.7	-0.0235	-0.0309	-0.0621	-0.1053	-0.1636
1.8	-0.0233	-0.0306	-0.0617	-0.1048	-0.1630
1.9	-0.0230	-0.0302	-0.0613	-0.1042	-0.1623
2.0	-0.0227	-0.0299	-0.0607	-0.1036	-0.1616
2.1	-0.0224	-0.0296	-0.0603	-0.1029	-0.1608
2.2	-0.0223	-0.0294	-0.0599	-0.1024	-0.1601
2.3	-0.0226	-0.0297	-0.0600	-0.1023	-0.1599
2.4	-0.0240	-0.0310	-0.0610	-0.1032	-0.1606
2.5	-0.0273	-0.0342	-0.0640	-0.1059	-0.1631
2.6	-0.0339	-0.0407	-0.0702	-0.1119	-0.1688
2.7	-0.0449	-0.0516	-0.0808	-0.1222	-0.1789
2.8	-0.0610	-0.0676	-0.0964	-0.1375	-0.1940
2.9	-0.0812	-0.0876	-0.1160	-0.1568	-0.2131
3.0	-0.1024	-0.1087	-0.1366	-0.1771	-0.2331
3.1	-0.1199	-0.1259	-0.1534	-0.1935	-0.2491
3.2	-0.1282	-0.1341	-0.1611	-0.2008	-0.2560
3.3	-0.1237	-0.1335	-0.1630	-0.2028	-0.2579
3.4	-0.1061	-0.1261	-0.1640	-0.2051	-0.2604
3.5	-0.0786	-0.1123	-0.1637	-0.2073	-0.2634
3.6	-0.0469	-0.0931	-0.1618	-0.2093	-0.2665
3.7	-0.0171	-0.0706	-0.1580	-0.2107	-0.2697
3.8	+0.0066	-0.0470	-0.1523	-0.2114	-0.2727
3.9	+0.0225	-0.0245	-0.1447	-0.2112	-0.2755
4.0	+0.0310	-0.0050	-0.1352	-0.2100	-0.2779
4.1	+0.0339	+0.0106	-0.1240	-0.2077	-0.2797
4.2	+0.0334	+0.0218	-0.1114	-0.2042	-0.2810
4.3	+0.0312	+0.0290	-0.0978	-0.1995	-0.2815
4.4	+0.0283	+0.0328	-0.0835	-0.1935	-0.2813
4.5	+0.0255	+0.0341	-0.0688	-0.1864	-0.2802
4.6	+0.0228	+0.0335	-0.0543	-0.1781	-0.2783
4.7	+0.0205	+0.0319	-0.0402	-0.1688	-0.2754
4.8	+0.0185	+0.0298	-0.0269	-0.1585	-0.2716
4.9	+0.0168	+0.0274	-0.0145	-0.1474	-0.2668
5.0	+0.0152	+0.0251	-0.0034	-0.1356	-0.2610
5.5	+0.0100	+0.0162	+0.0315	-0.0719	-0.2186
6.0	+0.0070	+0.0111	+0.0379	-0.0143	-0.1592
6.5	+0.0051	+0.0080	+0.0319	+0.0247	-0.0940
7.0	+0.0039	+0.0060	+0.0242	+0.0432	-0.0343
7.5	+0.0030	+0.0047	+0.0182	+0.0466	+0.0118
8.0	+0.0024	+0.0037	+0.0140	+0.0421	+0.0415
8.5	+0.0020	+0.0030	+0.0110	+0.0351	+0.0561
9.0	+0.0016	+0.0025	+0.0089	+0.0285	+0.0596
9.5	+0.0013	+0.0020	+0.0073	+0.0230	+0.0565
10.0	+0.0011	+0.0017	+0.0061	+0.0188	+0.0502

Table A3

The Values of  $\Delta f$  as a Function of  $\alpha$ .

r (earth radii)	Ring current field (gamma)			
	$\alpha = 1.0$	$\alpha = 2.0$	$\alpha = 3.0$	$\alpha = 4.0$
1.0	-0.2240	-0.1649	-0.1017	-0.0784
1.1	-0.2239	-0.1647	-0.1013	-0.0779
1.2	-0.2239	-0.1644	-0.1008	-0.0773
1.3	-0.2239	-0.1641	-0.1002	-0.0766
1.4	-0.2238	-0.1638	-0.0996	-0.0758
1.5	-0.2237	-0.1634	-0.0989	-0.0749
1.6	-0.2237	-0.1629	-0.0981	-0.0739
1.7	-0.2235	-0.1624	-0.0972	-0.0728
1.8	-0.2234	-0.1618	-0.0962	-0.0715
1.9	-0.2232	-0.1611	-0.0950	-0.0700
2.0	-0.2230	-0.1604	-0.0938	-0.0684
2.1	-0.0228	-0.1596	-0.0923	-0.0665
2.2	-0.0226	-0.1587	-0.0908	-0.0645
2.3	-0.2224	-0.1578	-0.0891	-0.0623
2.4	-0.2226	-0.1572	-0.0877	-0.0603
2.5	-0.2238	-0.1576	-0.0873	-0.0592
2.6	-0.2273	-0.1605	-0.0894	-0.0606
2.7	-0.2350	-0.1679	-0.0962	-0.0669
2.8	-0.2489	-0.1821	-0.1102	-0.0805
2.9	-0.2692	-0.2036	-0.1319	-0.1023
3.0	-0.2929	-0.2292	-0.1582	-0.1290
3.1	-0.3126	-0.2511	-0.1811	-0.1525
3.2	-0.3197	-0.2599	-0.1908	-0.1626
3.3	-0.3193	-0.2611	-0.1926	-0.1648
3.4	-0.3196	-0.2631	-0.1954	-0.1681
3.5	-0.3202	-0.2656	-0.1988	-0.1721
3.6	-0.3210	-0.2685	-0.2027	-0.1767
3.7	-0.3217	-0.2714	-0.2067	-0.1815
3.8	-0.3223	-0.2742	-0.2107	-0.1863
3.9	-0.3225	-0.2768	-0.2146	-0.1911
4.0	-0.3222	-0.2790	-0.2182	-0.1956
4.1	-0.3214	-0.2807	-0.2213	-0.1998
4.2	-0.3200	-0.2819	-0.2239	-0.2034
4.3	-0.3178	-0.2823	-0.2258	-0.2065
4.4	-0.3149	-0.2820	-0.2270	-0.2088
4.5	-0.3111	-0.2809	-0.2274	-0.2104
4.6	-0.3064	-0.2789	-0.2269	-0.2111
4.7	-0.3009	-0.2759	-0.2255	-0.2109
4.8	-0.2944	-0.2721	-0.2232	-0.2098
4.9	-0.2871	-0.2672	-0.2198	-0.2076
5.0	-0.2788	-0.2614	-0.2155	-0.2045
5.5	-0.2254	-0.2189	-0.1794	-0.1740
6.0	-0.1581	-0.1594	-0.1244	-0.1236
6.5	-0.0881	-0.0941	-0.0614	-0.0637
7.0	-0.0263	-0.0344	-0.0020	-0.0060
7.5	+0.0202	+0.0117	+0.0455	+0.0409
8.0	+0.0493	+0.0414	+0.0774	+0.0731
8.5	+0.0630	+0.0560	+0.0672	+0.0634
9.0	+0.0655	+0.0596	+0.0506	+0.0473
9.5	+0.0615	+0.0564	+0.0394	+0.0366
10.0	+0.0545	+0.0501	+0.0315	+0.0291